

Chapter 1

AN OVERVIEW OF HARMONICS MODELING AND SIMULATION

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1.1 Introduction

Distortion of sinusoidal voltage and current waveforms caused by harmonics is one of the major power quality concerns in electric power industry. Considerable efforts have been made in recent years to improve the management of harmonic distortions in power systems. Standards for harmonic control have been established. Instruments for harmonic measurements are widely available. The area of power system harmonic analysis has also experienced significant advancement [1,2]. Well-accepted component models, simulation methods and analysis procedures for conducting systematic harmonic studies have been developed. In this chapter we present an overview of the harmonics modeling and simulation issues and also provide an outline of this tutorial.

1.2 Fourier Series and Power System Harmonics

Fourier Series: The primary scope of harmonics modeling and simulation is in the study of periodic, steady-state distortion. The Fourier series for a regular, integrable, periodic function $f(t)$, of period T seconds and fundamental frequency $f=1/T$ Hz, or $\omega=2\pi f$ rad/s, can be written as [3]:

$$f(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega t + \theta_n) \quad (1.1)$$

where C_0 is the dc value of the function. C_n is the peak value of the n^{th} harmonic component and θ_n is its phase angle. A plot of normalized harmonic amplitudes C_n/C_1 is called the harmonic magnitude spectrum, as illustrated in Figure 1.1. The superposition of harmonic components to create the original waveform is shown in Figure 1.2.

Domain of Application: In general one can think of devices that produce distortion as exhibiting a nonlinear relationship between voltage and current. Such relationships can lead to several forms of distortion summarized as:

- A periodic steady-state exists and the distorted waveform has a Fourier series with fundamental frequency equal to power system frequency.
- A periodic steady state exists and the distorted waveform has a Fourier series with fundamental

frequency that is a sub-multiple of power system frequency.

- The waveform is aperiodic but can be expressed as a trigonometric series [3]. In this case the components in the Fourier series that are not integral multiples of the power frequency are sometimes called ‘non-integer’ harmonics.
- The waveform is aperiodic where the Fourier series is an approximation [4].

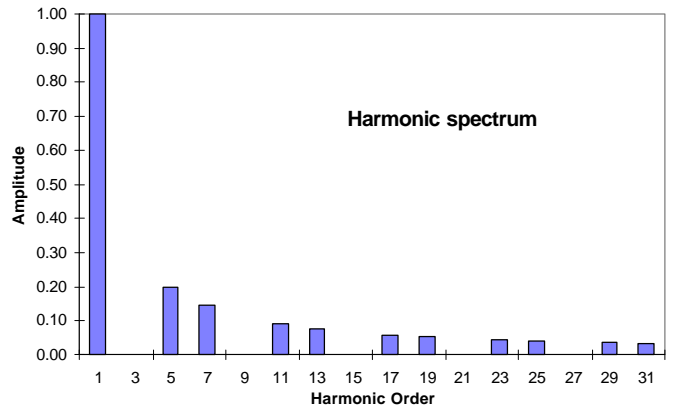


Figure 1.1. A harmonic (amplitude) spectrum.

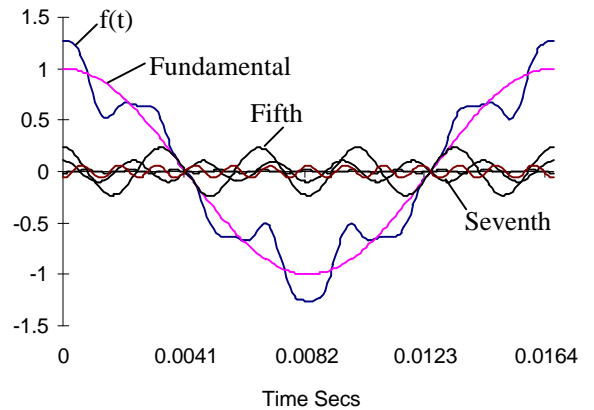


Figure 1.2. Synthesis of a waveform from harmonics.

The first case is commonly encountered and there are several advantages to using the decomposition in terms of harmonics. Harmonics have a physical interpretation and an intuitive appeal. Since the transmission network is usually modeled as a *linear* system, the propagation of each

harmonic can be studied independent of the others. The number of harmonics to be considered is usually small, which simplifies computation. Consequences such as losses can be related to harmonic components and measures of waveform quality can be developed in terms of harmonic amplitudes.

Certain types of pulsed or modulated loads create waveforms corresponding to the second category. The third category can occur in certain pulse-width modulated systems. Some practical situations such as arc furnaces and transformer inrush currents correspond to the fourth case. DC arc furnaces utilize conventional multiphase rectifiers but the underlying process of melting is not a stationary process. When reference is made to harmonics in this instance it corresponds to the periodic waveform that would be obtained if furnace conditions were to be maintained constant over a period of time. Harmonics modeling can lend insight into some of the potential problems but transient studies become very important.

The Origin of Harmonics: Main sources of harmonics in conventional power systems are summarized below.

1. Devices involving electronic switching: Electronic power processing equipment utilizes switching devices. The switching process is generally, but not necessarily, synchronized to the ac voltage.
2. Devices with nonlinear voltage-current relationships: Iron-core reactors and arcing loads are typical examples of such devices. When excited with a periodic input voltage the nonlinear v-i curve leads to the generation of harmonic currents.

Distortion Indices: The most commonly used measure of the quality of a periodic waveform is the total harmonic distortion (THD).

$$\text{THD} = \sqrt{\sum_{n=2}^{\infty} C_n^2} / C_1$$

(1.2)

IEEE Std. 519 [5] recommends limits on voltage and current THD values. Other indices such as telephone interference factor (TIF) and I•T product are used to measure telephone interference. The K-factor indices are used to describe the impact of harmonics on losses and are useful in de-rating equipment such as transformers.

Harmonics in Balanced and Unbalanced Three-Phase Systems: In balanced three-phase systems and under balanced operating conditions, harmonics in each phase have specific phase relationships. For example, in the case of the third harmonic, phase b currents would lag those in phase a by $3 \times 120^\circ$ or 360° , and those in phase c would lead by the same amount. Thus, the third harmonics have no phase shift and appear as zero-sequence components. Similar analysis shows that fifth harmonics appear to be of

negative sequence, seventh are of positive sequence, etc. System impedances must be appropriately modeled based on the sequences.

The magnitudes and phase angles (in particular) of three-phase harmonic voltages and currents are sensitive to network or load unbalance. Even for small deviations from balanced conditions at the fundamental frequency, it has been noted that harmonic unbalance can be significant. In the unbalanced case line currents and neutral currents can contain all orders of harmonics and contain components of all sequences. Three-phase power electronic converters can generate non-characteristic under unbalanced operation.

1.3 Harmonics Modeling and Simulation

The goal of harmonic studies is to quantify the distortion in voltage and current waveforms at various points in a power system. The results are useful for evaluating corrective measures and troubleshooting harmonic caused problems. Harmonic studies can also determine the existence of dangerous resonant conditions and verify compliance with harmonic limits. The need for a harmonic study may be indicated by excessive measured distortion in existing systems or by installation of harmonic-producing equipment. Similar to other power systems studies the harmonics study consists of the following steps:

- Definition of harmonic-producing equipment and determination of models for their representation.
- Determination of the models to represent other components in the system including external networks.
- Simulation of the system for various scenarios.

Many models have been proposed for representing harmonic sources as well as linear components. Various network harmonic solution algorithms have also been published. In the following sections, we briefly summarize the well-accepted methods for harmonic modeling and simulations. Other chapters in this tutorial will expand upon these ideas and illustrate how to set up studies in typical situations.

1.4 Nature and Modeling of Harmonic Sources

The most common model for harmonic sources is in the form of a harmonic current source, specified by its magnitude and phase spectrum. The phase is usually defined with respect to the fundamental component of the terminal voltage. The data can be obtained from an idealized theoretical model or from actual measurements. In many cases, the measured waveforms provide a more realistic representation of the harmonic sources to be modeled. This is particularly true if the system has significant unbalances or if non-integer harmonics are present. When a system contains a single dominant source of harmonics the phase spectrum is not important. However, phase angles must be

represented when multiple sources are present. A common method is to modify the phase spectrum according to the phase angle of the fundamental frequency voltage seen by the load. Ignoring phase angles does not always result in the ‘worst case’.

More detailed models become necessary if voltage distortion is significant or if voltages are unbalanced. There are three basic approaches that can be taken to develop detailed models:

- Develop analytical formulas for the Fourier series as a function of terminal voltage and operating parameters for the device.
- Develop analytical models for device operation and solve for device current waveform by a suitable iterative method.
- Solve for device steady state current waveform using time domain simulation.

Advanced models require design data for the device. For example, for a medium power ASD it is necessary to specify parameters such as transformer data, dc link data and motor parameters. Apart from potentially higher accuracy, an important advantage of such detailed models is that the user can specify operating conditions, e.g., motor speed in a drive, rather than spectra.

In the analysis of distribution and commercial power systems one may deal with a harmonic source that is an aggregate of many sources. Such a source can be modeled by measuring the aggregate spectrum. It is very difficult to develop a current source type model analytically based on the load composition data. Reference [7] has pointed out that the aggregate waveforms can be much less distorted than individual device waveforms.

Harmonic sources may also exhibit time-varying characteristics. Since standards and practice permit harmonic guidelines to be violated for short periods of time, including the time-varying characteristics of harmonic sources can be useful and can present a more realistic picture of actual distortions. More research is needed in this area [8].

Nonlinear Voltage-Current Sources: The most common sources in this category are transformers (due to their nonlinear magnetization requirements), fluorescent and other gas discharge lighting, and devices such as arc-furnaces. In all cases there exists a *nonlinear* relationship between the current and voltage. The harmonic currents generated by these devices can be significantly affected by the waveforms and peak values of supply voltages. It is desirable to represent the devices with their actual nonlinear v-i characteristics in harmonic studies, instead of as voltage independent harmonic current sources.

Power Electronic Converters: Examples of power electronic devices are adjustable speed drives, HVDC links, and static var compensators. Compared to the non-linear v-i devices, harmonics from these converters are less sensitive to supply voltage variation and distortion. Harmonic current source models are therefore commonly used to represent these devices. As discussed before, the phase angles of the current sources are functions of the supply voltage phase angle. They must be modeled adequately for harmonic analysis involving more than one source. The devices are sensitive to supply voltage unbalance. For large power electronic devices such as HVDC terminals and transmission level SVCs, detailed three-phase models may be needed. Factors such as firing-angle dependent harmonic generation and supply voltage unbalance are taken into account in the model. These studies normally scan through various possible device operating conditions and filter performance.

Rotating Machines: Rotating machines can be a harmonic source as well. The mechanism of harmonic generation in synchronous machines is unique. It cannot be described by using either the nonlinear v-i device model or the power electronic switching model. Only the salient pole synchronous machines operated under unbalanced conditions can generate harmonics with sufficient magnitudes. In this case, a unbalanced current experienced by the generator induces a second harmonic current in the field winding, which in turn induces a third harmonic current in the stator. In a similar manner, distorted system voltage can cause the machines to produce harmonics. Models to represent such mechanisms have been proposed [1]. For the cases of saturation-caused harmonic generation from rotating machines, the nonlinear v-i model can be used.

High frequency sources: Advances in power electronic devices have created the potential for a wide range of new power conversion techniques. The electronic ballast for fluorescent lighting is one example. In general, these systems employ high frequency switching to achieve greater flexibility in power conversion. With proper design, these techniques can be used to reduce the low frequency harmonics. Distortion is created at the switching frequency, which is generally above 20 kHz. At such high frequency, current distortion generally does not penetrate far into the system but the possibility of system resonance at the switching frequency can still exist.

Non-integer harmonic sources: There exist several power electronic systems which produce distortion at frequencies that are harmonics of a base frequency other than 60 Hz. There are also devices that produce distortion at discrete frequencies that are not integer multiples of the base frequency. Some devices have waveforms that do not submit to a Fourier or trigonometric series representation. Lacking standard terminology, we will call these non-harmonic sources. Modeling of this type of harmonic sources has attracted many research interests recently.

1.5 Network and Load Models

Network Model: The main difficulty in setting up a network model is to determine *how much of the network* needs to be modeled. The extent of network representation is limited by available data and computing resources. The following observations can be made:

- For industrial power systems connected to strong or dedicated three-phase distribution feeders it is generally sufficient to model two transformations from the load point. Generally, transformer impedances dominate. Branch circuits should be modeled if they connect to power factor correction capacitors or motors. Although capacitance of overhead lines is usually neglected, cable capacitance should be modeled for cables longer than 500 feet.
- Large industrial facilities are served at sub-transmission and even transmission voltage. In this case it is important to model at least a portion of the HV/EHV network if the facility has multiple supply substations. If it has only one supply substation, utilities may provide the driving-point impedance seen by the facility.
- Distribution feeders (at least in the US and Canada) are unbalanced and loads are often served from single phase laterals. Shunt capacitors are extensively used. Thus it becomes mandatory to model the entire feeder, and sometimes adjacent feeders as well.

The above observations are not guaranteed rules, but are based on common practice. Perhaps the best way to determine the extent of network modeling needed is to perform a sensitivity study; i.e., one can progressively expand the network model until the results do not change significantly. In many harmonic studies involving industrial plants, the supply system is represented as a frequency-dependent driving-point impedance at the point of common coupling.

Overhead Lines and Underground Cables: Modeling of lines and cables over a wide range of frequencies is relatively well documented in literature [9]. Typical lines or cables can be modeled by multiphase coupled equivalent circuits. For balanced harmonic analysis the models can be further simplified into single-phase pi-circuits using positive and zero sequence data. The main issues in modeling these components are the frequency dependence of per-unit length series impedance and the long line effects. As a result, the level of detail of their models depends on the line length and harmonic order:

- In industrial systems and utility distribution systems where line lengths are short it is customary to use sequence impedances. Capacitance is usually neglected except in the case of long cable runs.

- An estimate of line-length beyond which long line models should be used is $150/n$ miles for overhead line and $90/n$ miles for underground cable, where n is the harmonic number.
- Skin effect correction is important in EHV systems because line resistance is the principal source of damping.

Transformers: In most applications, transformers are modeled as a series impedance with resistance adjusted for skin effects. This is because adequate data is usually not available. Three-phase transformer connections may provide $\pm 30^\circ$ phase shift. Other connections such as zigzag windings are used to mitigate harmonics. The phase shifts associated with transformer connections must be accounted for in multiple source systems.

Other considerations include the nonlinear characteristics of core loss resistance, the winding stray capacitance and core saturation. Harmonic effects due to nonlinear resistance are small compared to the nonlinear inductance. Effects of stray capacitance are usually noticeable only for frequencies higher than 4 kHz. The saturation characteristics can be represented as a harmonic source using the nonlinear v-i model if saturation-caused harmonic generation is of concern.

Passive Loads: Linear passive loads have a significant effect on system frequency response primarily near resonant frequencies. As in other power system studies it is only practical to model an aggregate load for which reasonably good estimates (MW and MVAR) are usually readily available. Such an aggregate model should include the distribution or service transformer. At power frequencies the effect of distribution transformer impedance is not of concern in the analysis of the high voltage network. At harmonic frequencies the impedance of the transformer can be comparable to that of motor loads, because induction motors appear as locked-rotor impedances at these frequencies.

A general model thus appears as in Figure 1.3. To characterize this model properly, it is necessary to know the typical composition of the load. Such data are usually not easily available. The following models have been suggested in literature (n represents the harmonic order):

Model A : Parallel R,L with $R = V^2 / (P)$; $L = V^2 / (2\pi f Q)$

This model assumes that the total reactive load is assigned to an inductor L . Because a majority of reactive power corresponds to induction motors, this model is not recommended.

Model B : Parallel R,L with
 $R = V^2 / (k \cdot P)$, $L = V^2 / (2\pi f k \cdot Q)$; $k = .1h + .9$

Model C : Parallel R,L in series with transformer inductance L_s , where

$$R = V^2/P; L = n R/(2\pi f 6.7*(Q/P)-.74); L_s = .073 \text{ h R}$$

Model C is derived from measurements on medium voltage loads using audio frequency ripple generators. The coefficients cited above correspond to one set of studies [10], and may not be appropriate for all loads. Load representation for harmonic analysis is an active research area.

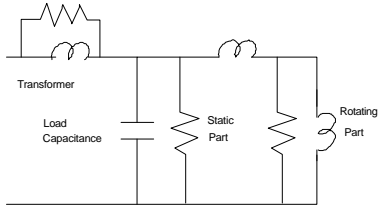


Figure 1.3: Basic Load Model.

Large Rotating Loads: In synchronous and induction machines the rotating magnetic field created by a stator harmonic rotates at a speed significantly different from that of the rotor. Therefore at harmonic frequencies the impedance approaches the negative sequence impedance. In the case of synchronous machines the inductance is usually taken to be either the negative sequence impedance or the average of direct and quadrature sub-transient impedances. For induction machines the inductance is taken to be the locked rotor inductance. In each case the frequency-dependence of resistances can be significant. The resistance normally increase in the form n^a where n is the harmonic order and the parameter ‘ a ’ ranges from 0.5-1.5. Most motors are delta-connected and therefore do not provide a path for zero-sequence harmonics.

1.6 Harmonic Simulation

It is appropriate to note that a large number of harmonic related problems encountered in practice involve systems with relatively low distortion and often a single dominant harmonic source. In these cases simplified resonant frequency calculations, for example, can be performed by hand [5] and distortion calculations can be made with a simple spreadsheet. For larger systems and complicated harmonic producing loads, more formal harmonic power flow analysis methods are needed. In this section, techniques presently being used for harmonics studies are reviewed. These techniques vary in terms of data requirements, modeling complexity, problem formulation, and solution algorithms. New methods are being developed and published.

Mathematically, the harmonic study involves solving the network equation for each harmonic written in matrix form as

$$[\mathbf{I}_m] = [\mathbf{Y}_m][\mathbf{V}_m] \quad m=1, 2 \dots n \quad (1.3)$$

where $[\mathbf{Y}_m]$ represents the nodal admittance matrix, $[\mathbf{I}_m]$ is the vector of source currents and $[\mathbf{V}_m]$ is the vector of bus voltages for harmonic number m . In more advanced approaches the current source vector becomes a function of bus voltage.

Frequency Scan: The frequency scan is usually the first step in a harmonic study. A frequency or impedance scan is a plot of the driving point (Thevenin) impedance at a system bus versus frequency. The bus of interest is one where a harmonic source exists. For simple system this impedance can be obtained from an impedance diagram. More formally, the Thevenin impedance can be calculated by injecting a 1 per unit source at appropriate frequency into the bus of interest. The other currents are set to zero and (1.3) is solved for bus voltages. These voltages equal the driving-point and transfer impedances. The calculation is repeated over the harmonic frequency range of interest. Typically, a scan is developed for both positive and zero sequence networks.

If a harmonic source is connected to the bus of interest, the harmonic voltage at the bus is given by the harmonic current multiplied by the harmonic impedance. The frequency scan thus gives a visual picture of impedance levels and potential voltage distortion. It is a very effective tool to detect resonances which appear as peaks (parallel resonance) and valleys (series resonance) in the plot of impedance magnitude vs. frequency.

Simple Distortion Calculations: In the simplest harmonic studies harmonic sources are represented as current sources specified by their current spectra. Admittance matrices are then constructed and harmonic voltage components are calculated from (1.3). The harmonic current components have a magnitude determined from the typical harmonic spectrum and rated load current for the harmonic producing device.

$$I_n = I_{\text{rated}} I_{n\text{-spectrum}} / I_{1\text{-spectrum}}$$

where n is the harmonic order and the subscript ‘spectrum’ indicates the typical harmonic spectrum of the element. To compute indices such as THD the nominal bus voltage is used.

For the multiple harmonic source cases it is important to also model the phase angle of harmonics. A fundamental frequency power-flow solution is needed, because the harmonic phase angles are functions of the fundamental frequency phase angle as follows:

$$\theta_n = \theta_{n\text{-spectrum}} + n(\theta_1 - \theta_{1\text{-spectrum}})$$

where θ_1 is the phase angle of the harmonic source current at the fundamental frequency. $\theta_{n\text{-spectrum}}$ is the phase angle of the n-th harmonic current spectrum. Depending on the phase angles used, the effects of multiple harmonic sources can either add or cancel. Ignoring phase relationships may, therefore, lead to pessimistic or optimistic results.

Harmonic Power Flow Methods: The simple distortion calculation discussed above is the basis for most harmonic study software and is useful in many practical cases. The main disadvantage of the method is the use of ‘typical’ spectra. This prevents an assessment of non-typical operating conditions. Such conditions include partial loading of harmonic-producing devices, excessive distortion and unbalance. To explore such conditions the user must develop typical spectra for each condition when using the simplified method. The disadvantages have prompted the development of advanced harmonic analysis methods. The goal is to model the physical aspects of harmonic generation from the device as a function of actual system conditions.

The general idea is to create a model for the harmonic producing device in the form

$$F(V_1, V_2, \dots, V_n, I_1, I_2, \dots, I_n, C) = 0 \quad (1.4)$$

Here V_1, V_2, \dots, V_n are harmonic voltage components, I_1, I_2, \dots, I_n , are corresponding harmonic current components and C represents multiple operating and design parameters. Equation (1.4) permits the calculation of harmonic currents from voltages and includes power flow constraints. The total procedure is to simultaneously solve (1.3) and (1.4).

One of the well-known methods is the so called “harmonic iteration method” [11,12]. Equation (1.4) is first solved using an estimated supply voltage. The resulting current spectrum is used in (1.3) to calculate the supply voltage. This iterative process is repeated until convergence is achieved. Reliable convergence is achieved although difficulties may occur when sharp resonances exist. Convergence can be improved by including a linearized model of (1.4) in (1.3). A particular advantage of this “decoupled” approach is that device models in the form of (1.4) can be in a closed form, a time domain model, or in any other suitable form.

Another method is to solve (1.3) and (1.4) simultaneously using Newton type algorithms. This method requires that device models be available in closed form wherein derivatives can be efficiently computed [13]. The various methods above can be extended, with a significant increase in computational burden, to the unbalanced case. Both (1.3) and (1.4) are cast in a multi-phase framework [11,14]. Such an approach can have several advantages. The first is the modeling of zero

sequence current flow. Second is the capability of addressing non-characteristic harmonics.

Finally, it is appropriate to note that harmonic studies can be performed in the time domain. The idea is to run a time-domain simulation until a steady state is reached. The challenge is first to identify that a steady-state has indeed been achieved. Secondly, in lightly damped systems techniques are needed to obtain the steady-state conditions within a reasonable amount of computation time. References [14,15] provide examples of such methods.

1.6 Summary

Harmonic studies are becoming an important component of power system planning and design. In using software to analyze practical conditions it is important to understand the assumptions made and the modeling capabilities. Models and methods used depend upon system complexity and data availability. The purpose of this tutorial is to suggest what is required to set up harmonics studies with emphasis on modeling and simulation.

This overview has attempted to summarize key ideas from chapters that follow. The propagation of harmonic current in a power system, and the resulting voltage distortion, depends on the characteristics of harmonic sources as well as the frequency response of system components. Characteristics of various harmonic sources and consideration in their modeling have been summarized. Component modeling has been described. Different approaches to conduct analysis were discussed in a common framework. Subsequent chapters of this tutorial will expand upon each of these topics and provided illustrative examples.

Acknowledgments

This chapter was adapted from a paper developed by the Task Force on Harmonics Modeling and Simulation [1].

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