MODELING OF HARMONIC SOURCES

POWER ELECTRONIC CONVERTERS

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4.1 Introduction

Harmonic problems are not new to electric utility and industrial power systems. In the past, most harmonic-related problems were caused by large nonlinear loads such as arc furnaces. These types of problems have been effectively mitigated. However, due to the widespread proliferation of power electronic controlled devices nowadays, the problems caused by harmonics are of increasing importance. Power electronic loads offer a number of advantages in controlling power flow and in efficiency, but they perform this by chopping, flatting, or shaping sinusoidal voltages and currents. Harmonics are produced in the process.

Among today's power electronic applications, most of the harmonic problems are caused by the static power converters. The static converters are used in many types of industrial applications. The purpose of this chapter is to present modeling and simulation techniques for power electronic devices, focusing on the harmonic modeling for static power converters. First, we briefly review the commonly seen power electronic type harmonic sources. Next, a number of major converter harmonic models used in harmonic simulation will be described. Two converter models used as harmonic simulation examples are then presented.

4.2 Review of Power Electronic Harmonic Sources

Due to the advanced technologies in power electronics development over the past decade, the application of power electronics has been widely spread to all types of industries. Commonly observed examples are:

Line Commutated Converters

The introduction of economic and reliable line commutated converters has caused a significant increase in harmonic-generating loads, and they have dispersed over the entire power system. In most cases, line commutated converters are the cause of harmonic problems in power distribution systems. These devices are work horse circuits for ac/dc power conversion. The common application of static power converters is in adjustable speed drives for motor control. Another application is in HVDC terminals. The device can be operated as a six-pulse converter, as shown in Figure 4.1, or configured in parallel arrangements for higher pulse operation. Theoretically, a static power converter load draws currents from the source system that consist of positive and negative currents which are equally separated. The pulse number refers to the number of "humps" on the dc output voltage that are produced during every ac cycle.

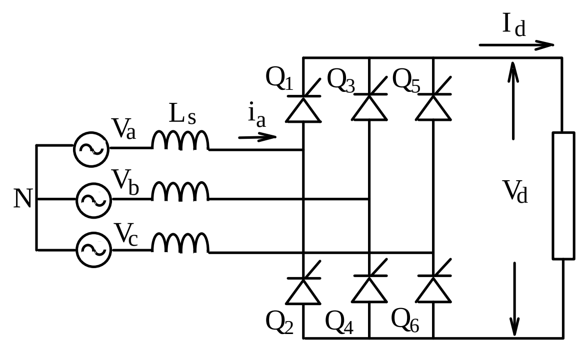

Figure 4.1. Six-Pulse Line Commutated Converter

In Figure 4.1, each pair of thyristors is triggered (firing angle) and conduct until they are reverse-biased. If a thyristor is triggered at zero firing angle, it acts exactly like a diode. The term line commutated converter refers to the fact that the load actually turns thyristors off, rather than them being turned off by external control circuits. The ideal ac current waveform for a six-pulse converter is on for 120 degrees and off for another 60 degrees. During the on period, the dc load current is assumed constant in the ideal case due to the assumed existence of a large series dc inductor. Assuming no commutation overlap and balanced three-phase operation, it can be shown that the phase a current is

$$i_a(t) = \sum_h \frac{I_1}{h} \sin(h\omega_1 t + \delta_h), \qquad (4.1)$$

where h = 1, 5, 7, 11, 13, ... We see that the ac harmonic currents generated by a six-pulse converter include all odd harmonics except triplens. Harmonics generated by converters of any pulse number can be expressed by

$h = pn \pm 1$, where n is any integer and p is the pulse number of the converter. For the ideal case, converter harmonic current magnitudes decrease according to *1/h* rule. Table 4.1 gives the *1/h*-rule and typical harmonic currents (in per unit of the fundamental component) for six-pulse converters [1].

Table 4.1. Theoretical and Typical Harmonic Currents for Six-Pulse Converters.

h	5	7	11	13	17	19	23	25
1/h-rule	.200	.143	.091	.077	.059	.053	.043	.040
Typical	.175	.111	.045	.029	.015	.010	.009	.008

Pulse-Width Modulated Converters

PWM converters use power electronic devices that can be turned off and turned on. Therefore, voltage and current waveforms can be shaped more desirably. The switching components can be thyristors that are forced off by external control circuits, or they can be GTOs or power transistors. The latter devices are usually used because of their fast switching characteristics are needed for effective PWM.

In a PWM converter, the switching devices are controlled to switch on and off to produce a series of pulses. These pulses are to be varied in width to produce a pulsed three-phase voltage wave for the load. Due to their low efficiencies, PWM converters are limited to low power applications in the several hundred kW or hp ranges.

Cycloconverters

The cycloconverter is a device that converts ac power at one frequency into ac power at a lower frequency. Cycloconverters are usually used in low speed and large horsepower applications. The harmonic frequencies generated by a cycloconverter depend on the output frequency, which is varied in operation to control motor speed. The output frequency of a cycloconverter can be controlled by precisely timing the firing pulses at its thyristor gates through computer control.

Static VAR Compensator (SVC)

The static var compensator is used as a voltage controller in the power system. This device controls network voltage by adjusting the amount of reactive power supplied to or absorbed from the power system. The applications of the SVC are usually for local compensation of reactive power to industrial loads and for regulation of utility network voltages to improve transfer capabilities across the transmission system. Typical configuration of an SVC consists of shunt capacitors with a thyristor-controlled reactor (TCR) connected in parallel.

Other Power Electronic Devices

Other power electronic devices which may generate harmonics in the power system include static phase shifters, isolation switches, load transfer switches, and energy storage and instantaneous backup power systems as well as those devices covered under the subjects of Flexible AC Transmission System (FACTS) and Custom Power Systems (CPS) [2].

4.3 Review of Static Power Converter Models for Harmonic Simulation

In order to simulate the propagation of harmonics throughout a network, adequate models for harmonic-generating loads as well as system components must be developed. In general, the power electronic devices that generate harmonic currents can be modeled by using simple current source models or complicated device-level models. The harmonic simulation can be in frequency domain, in time domain, or in both. In this section, an overview of common harmonic modeling techniques for static power converters for simulation studies will be described.

Power electronic converters for harmonic analysis can be simply represented by a harmonic current source or a model that takes into account the interaction between ac system network and the converter dc system. When the latter situation is considered, a more sophisticated converter analysis for the resulting harmonic currents as a function of system reactance, delay angle, and commutation angle is required. The accuracy of converter model needs also to be considered to guarantee the convergence of the simulation. At present, there are several techniques that have been developed for modeling of power electronic converters in harmonic simulation. These techniques can be categorized as:

1. Current injection model.
2. Frequency- or time-domain Norton equivalent circuit model.
3. Harmonic coupling matrix model.
4. Time- or frequency-domain device model used with frequency-domain network model.
5. Time-domain model.

The following sections give a brief overview on the aforementioned converter models for harmonic simulations. For the details of these models, please refer to the corresponding references.

Current Injection Model

The most common technique for harmonic simulation is to treat static power converters as known sources of harmonic currents with or without including phase angle information. This is due to the fact that the converter acts as an injection current source to the system in many operational conditions. Generally, the steady-state condition is assumed. The following frequency-domain matrix equations for each harmonic are used to compute the network harmonic voltages:

$$\mathbf{V_h} = \mathbf{Z_h} \cdot \mathbf{I_h} \tag{4.2}$$

or

$$\mathbf{I_h} = \mathbf{Y_h} \cdot \mathbf{V_h} \ . \tag{4.3}$$

Then, superposition is applied to convert the solved values of each $\mathbf{V_h}$ into the time domain for each network bus k as follows:

$$v_k(t) = \sum_{h=1}^{H} V_h^k \sin(h\omega_1 t + \delta_h^k) \,, \tag{4.4}$$

where H is the highest harmonic order under consideration.

In the current injection model, the magnitudes of harmonic currents can be determined simply following the *1/h* rule as stated in (4.1) or represented by measurements, as shown in Table 1. The phase angles of the current sources are functions of the supply voltage phase angle [3] and can be expressed as

$$\theta_h = \theta_{h-spec} + h(\theta_1 - \theta_{1-spec}) \,, \tag{4.5}$$

where θ_1 is the phase angle obtained from the load flow solution for fundamental frequency current component, and θ_{h-spec} is the typical phase angle of the harmonic source current spectrum. Many times, especially for studies involving one converter, the phase angles are ignored and only the magnitudes are used in the harmonic simulation. Once the harmonic voltages are known, harmonic currents through network elements are determined, and they can be converted to time-domain currents.

The advantages of the current injection method are that the solution can always be obtained directly (non-iterative) and it is computationally efficient. Ideally, this method is able to handle several harmonic sources simultaneously. The drawback of this method is that typical harmonic spectra are often used to represent the harmonic currents generated by the converter which ignores the interaction between the network and the converter. This prevents an adequate assessment of cases involving non-typical operating modes, such as partial loading, excessive harmonic voltage distortions and unbalanced network conditions. Reference [4] suggests that the current injection model should be used carefully (if at all) when the converter source voltage THD is on the order of 10% or more. More information on the current injection method and associated models can be found in [4-6].

The aforementioned drawbacks can be overcome by using more advanced converter models and harmonic analyses described in the following sections. The models generally include the effects of harmonic voltages on the converter current waveform. Therefore, these advanced methods couple the converter with the system admittance matrix, such as shown in (4.3), or some other more complicated expression of the power system. Given an initial estimate of harmonic current injections at the converter, the network bus harmonic voltages are determined. A new estimate of the harmonic injection currents is then obtained from the computed harmonic voltages. This process is repeated until convergence in the magnitude of the harmonic voltages on each network bus is reached.

<u>Frequency- or Time-Domain Norton Equivalent Circuit Model</u>

In this model, the converter is represented by a Norton equivalent circuit, where the Norton admittance represents an approximation of the converter response to variation in its terminal voltage harmonics. A common approach for this model to have the converter switching represented by a switching function whose frequency-domain expression is known [7]. The switching function is used to determine the ac side harmonic phasors directly from the dc side harmonic phasors. This model is then iteratively improved in a frequency-domain network solution process. This type of solution process in harmonic power flow analysis is usually called iterative harmonic analysis (IHA) [8]. Reference [9] also presents a similar model in time domain with an iterative simulation technique. More information on this model can be found in [10] and [11].

The advantages of the Norton equivalent model are that the solution process has better convergence characteristics and that a direct solution for the interaction between the converter and the network can be obtained. However, [12] reports that the problem with this model is that the converter is indeed an interface between the ac and dc systems, with only the ac system represented in the entire iterative solution process. If the converter controller needs to be modeled, a separate iterative process is required for solving the converter interaction with the dc system at each iteration.

<u>Harmonic Coupling Matrix Model</u>

In [13] and [14], the authors proposed an efficient technique by the linearization of the interaction between the converter dc system and the ac network. Then, the entire system is solved via the harmonic coupling matrix equation to account for the interaction between the converter dc system and the ac terminal voltage. Figure 4.2 shows a simplified single- or three-phase converter model, where the harmonic coupling matrix equation can expressed as

$$\begin{bmatrix} I_{ac} \\ V_{dc} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{ac} \\ I_{dc} \end{bmatrix} . \tag{4.6}$$

In (4.6), I_{ac} and V_{ac} can be expanded to include both positive and negative sequence components, and I_{dc} can be expanded to include the firing angle controls [13].

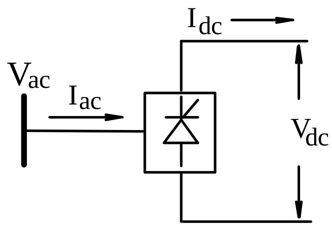

Figure 4.2. Simplified Converter Model

The harmonic coupling matrix provides a relationship between the harmonic components of ac side voltage/ current and the dc side voltage/current of the converter. Each element in the matrix is a sub-matrix and is a function of the converter states and commutating inductance. This model can be used either in the time or the frequency domain with the incorporation of the iterative approach, and it has been developed for both single-phase and three-phase converters while ignoring the effects of converter controls, commutation variations, and resistance in ac network impedance [12].

Time- or Frequency-Domain Device Model used with Frequency-Domain Network Model

In this model, the converter is described in terms of the actual time-domain differential equations that govern its performance. Then, converter currents are solved in the time-domain and converted into the frequency-domain by the use of Fourier analyses. Next, the harmonic currents are injected into the network model and the harmonic voltages at each network bus are calculated. The computed voltages are then used to recalculate the converter currents in the time domain. In Newton-Raphson or Gauss-Seidel types of harmonic power flow analysis, this procedure iterates until convergence criteria are met. The HARMFLO and HARMFLO+ computer programs are well-known products that use the combinations of time- and frequency-domain solutions. More details about this model can be found in [15-17].

Reference [18] also presents a frequency-domain model which formulates a general set of non-linear equations to describe the converter in steady state. The formulation convolutes periodic sampled quantities in the frequency domain with square pulse sampling functions. The use of sampling functions in this manner is similar to other work using the switching function [7]. The non-linear equations are then solved using Newton's method in conjunction with the frequency-domain network model.

Time-Domain Model

In the time-domain model, the solution method used is a time simulation of the entire system (both the converter and the ac network). These solution methods are the most mature of harmonic simulations. The programs such as EMTP, ATP, and EMTDC can be used to obtain a complete time-domain solution. The actual periods of operation within each cycle of converter operation are described by differential equations. No attempt is made to convert to the frequency domain. Both balanced and unbalanced conditions can be handled, and the converter model can be as detailed as necessary. However, the solution time and engineering effort increase significantly. References [19] and [20] also provide other insights for the time-domain model.

4.4 Case Study

In this section, we evaluate two converter models used in harmonic simulation. The two models are current injection model and harmonic Norton equivalent circuit model. A commonly seen PWM type adjustable speed drive (ASD) is chosen for evaluation. An ASD mainly consists of a converter (rectifier or front-end), a dc link, a controller, and an inverter. Generally, the harmonics produced in the inverter part are negligible as seen from the converter ac side because of the harmonic current path formed by the dc link capacitor. Therefore, the converter is modeled as the only part that injects harmonic currents into the power system for the PWM type ASD. Figure 4.3 shows the converter circuit of the ASD, where the inverter and the motor load are modeled as a direct current source.

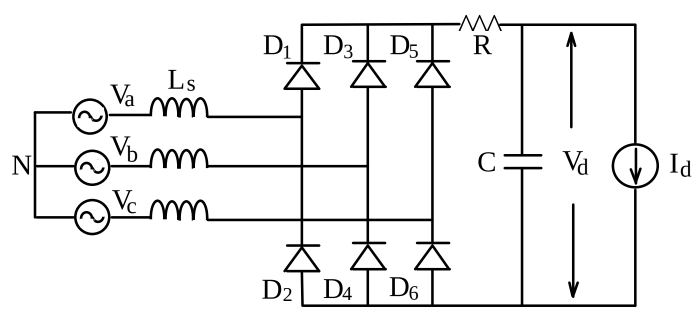

Figure 4.3. Converter Circuit Model of the PWM ASD

Norton Equivalent Circuit Model

The converter circuit shown in Figure 4.3 is solved with the harmonic analysis technique described in [7]. The end result is a delta connected Norton equivalent circuit. The circuit is then interfaced with the supply network in an iterative fashion, as described in [11], to determine the harmonic current injections from the ASD. Parameters needed to run the model are 1) the firing angle of the converter thyristors, α; 2) the direct current flowing into the inverter, I_d; and 3) the dc link R, L, and C component values.

The firing angle of the PWM type ASD is almost zero because of the use of diodes as the front end. The direct current flows into the inverter can be estimated from the motor load as

$$I_d = \frac{P}{2.34 V_g \cos\alpha} \quad , \qquad (4.7)$$

where P is the motor load including the losses, and Vg is the line to ground voltage of the supply system.

In a typical harmonic study, α and I_d need to be varied for investigation of the various ASD-motor operating conditions. Also, representation of the dc link is essential for the correct harmonic simulation. If the dc link parameters are not available, a simplified model such as the current injection model may be proposed.

Current Injection Model

An ASD may be represented as a harmonic current source. Table 4.3 gives the typical harmonic magnitude and phase spectra that can be used to model an ASD. The corresponding waveforms are shown in Figure 4.4. These data are obtained from the Norton equivalent model that simulates an actual PWM ASD and are verified by lab tests. Extensive analytical and numerical studies indicate that the data is suitable for modeling PWM type ASDs.

Table 4.3. Typical Harmonic Spectra of PWM Type ASDs

Load Level	100%		75%		50%	
h-order	Mag.	Angle	Mag.	Angle	Mag.	Angle
1	100.00	0	100.00	0	100.00	0
3	0.35	-159	0.59	-44	0.54	-96
5	60.82	-175	69.75	-174	75.09	-174
7	33.42	-172	47.03	-171	54.61	-171
9	0.50	158	0.32	-96	0.24	-102
11	3.84	166	6.86	17	14.65	16
13	7.74	-177	4.52	-178	1.95	71
15	0.41	135	0.37	-124	0.32	28
17	1.27	32	7.56	9	9.61	10
19	1.54	179	3.81	9	7.66	16
21	0.32	110	0.43	-163	0.43	95
23	1.08	38	2.59	11	0.94	-8
25	0.16	49	3.70	10	3.78	7

To use this model, the ASD is first represented as a constant power load at the fundamental frequency. The real power load is equal to the ASD/motor load and the reactive power load is zero. The network is then solved at the fundamental frequency. The ASD current magnitude and phase angle are determined as I_{60} and θ_{60}, respectively. The harmonic current source representing the ASD is calculated by scaling up the magnitude column of Table 4.3 by I_{60} and by shifting the phase angle column by $h\theta_{60}$. It can be seen that the current source model is easy to use and needs less input effort. The disadvantages are that the model cannot simulate a wide variety of ASDs and ASD/system operating conditions.

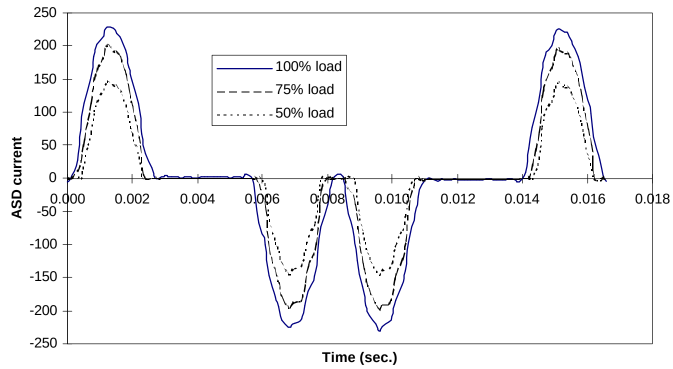

Figure 4.4. Typical Current Waveforms of PWM Type ASDs

Model Verification

Lab tests are conducted to verify the three-phase ASD model developed in this study. The tests are performed on a 30 hp PWM-type ASD serving a 20 hp induction motor load. The lab setup is shown in Figure 4.5. The ASD supply voltage (line to line), Vs, is 600 V. The motor mechanical load is varied to simulate different operating conditions. 13 operating conditions are recorded. For each operating condition, waveforms of 15 voltage and current quantities are measured when the system is in steady-state. These waveform snapshots, including source side voltages and currents, motor side voltages and currents, dc link voltage and inverter current, are synchronised and sampled at a rate of 100 kHz. In addition to the above operating tests, the dc link is measured at various frequencies in a standstill test to determine its component parameters.

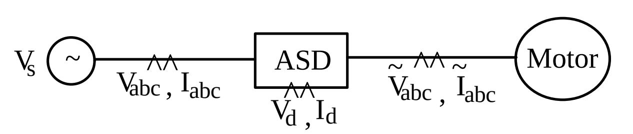

Figure 4.5. Lab Test Setup for ASD Harmonic Measurement

The ASD equivalent circuit model can be verified by comparing the measured ASD currents into the supply system against the calculated ones. Waveform comparison is more desirable than harmonic spectrum comparison because the former ensures that both the harmonic magnitudes and phase angles are checked. A network model representing the test conditions is constructed. The up-stream supply system is modelled as known three-phase harmonic voltage sources determined from the measured data.

Figure 4.6 gives the comparison for the three-phase ASD currents between measured and calculated values under one representative operating condition. In the figure, the measured

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Figure 4.6. Comparison of Measured and Calculated Waveforms

waveforms are plotted as solid lines while the computed waveforms are plotted as dashed lines. Some discrepancies between the measured and calculated waveforms are observed. A few factors may have contributed to the discrepancies. The first factor is that the size of ASD is relatively small. This makes the variations of thyristor electric characteristics more noticeable in the waveforms. The high frequency components of dc link current generated by the PWM scheme and inaccuracy of dc link parameters may also contribute to discrepancies. Tests showed that the waveforms obtained with an inductor added between the drive and the motor are in a closer agreement with the calculated ones. In addition, it is noticed that harmonics in the supply voltage have some impact on the calculated waveforms. Unfortunately, including more harmonics does not enhance the agreement. Overall, the agreement between the measured and the calculated waveforms show that the proposed models are acceptable.

4.5 Summary

The common models of power electronic converters used in harmonic analysis are presented in this chapter. These models are built either in time-domain or frequency-domain and in conjunction with the proper power system network model. Once built, the model is ready for use in a harmonic simulation technique such as non-iterative or iterative analysis.

The intention of this chapter is to give an overview on these converter models. A case study with two selected converter models is presented to demonstrate the harmonic simulation procedure and the accuracy of the proposed models. For more detailed model approaches, the reader is advised to look into the references provided here or in other sources. References [21-24] also provide harmonic models for other types of power electronic devices.

References

1. R. P. Stratford, “Analysis and Control of Harmonic Current in Systems with Static Power Converters,” *IEEE Trans. on Industry Applications*, Vol. IA-17, No. 1, January/February 1981, pp. 71-78.
2. IEEE Power Electronics Modeling Task Force & Digital Simulation Working Group, “Guidelines for Modeling Power Electronics in Electric Power Engineering Applications,” *IEEE Trans. on. Power Delivery*, Vol. 12, No. 1, January 1997, pp. 505-514.
3. Task Force on Harmonics Modeling and Simulation, “The Modeling and Simulation of the Propagation of Harmonics in Electric Power Networks Part I: Concepts, Models and Simulation Techniques,” *IEEE Trans. on Power Delivery*, Vol. 11, No. 1, January 1996, pp. 452-465.
4. M. F. McGranaghan, R. C. Dugan, and W. L. Sponsler, “Digital Simulation of Distribution System Frequency-Response Characteristics,” *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-100, No. 3, March 1981, pp. 1362-1369.
5. D. J. Pileggi, N. H. Chandra, and A. E. Emanuel, “Prediction of Harmonic Voltages in Distribution Systems,” *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-100, No. 3, March 1981, pp. 1307-1315.
6. T. H. Ortmeyer, “Harmonic Analysis Methodology,” *IEEE PES Tutorial Course, Course Text 84 EH0221-2-PWR*, February, 1984, pp. 74-84.
7. W. Xu, J. E. Drakos, Y. Mansour, and A. Chang, “A Three-Phase Converter Model for Harmonic Analysis of HVDC Systems,” *IEEE Trans. on Power Delivery*, Vol. 9, No. 3, July 1994, pp. 1724-1731.

8. J. Arrillaga and C. D. Callaghan, “Three Phase AC-DC Load and Harmonic Flows,” *IEEE Trans. on Power Delivery*, Vol. 6, No. 1, January 1991, pp. 238-244.
9. J. Usaola and J. G. Mayordomo, “Fast Steady-State Techniques for Harmonic Analysis,” *Proceedings of the 4th International Conference on Harmonics in Power Systems (ICHPS IV)*, Budapest, Hungary, October 1990, pp. 336-342.
10. B. C. Smith, N. R. Watson, A. R. Wood, and J. Arrillaga, “A Solution for the Steady-State Interaction of the AC/DC Converter with Weak AC and DC Systems,” *Proceedings of the 7th International Conference on Harmonics and Quality of Power (ICHQP VII)*, Las Vegas, NV, October 1996.
11. W. Xu, J. R. Jose and H. W. Dommel, “A Multiphase Harmonic Load Flow Solution Technique,” *IEEE Trans. on Power Systems*, Vol. PS-6, February 1991, pp. 174-182.
12. B. C. Smith, J. Arrillaga, A. R. Wood, and N. R. Watson, “A Review of Iterative Harmonic Analysis for AC-DC Power Systems,” *Proceedings of the 7th International Conference on Harmonics and Quality of Power (ICHQP VII)*, Las Vegas, NV, October 1996.
13. E. V. Larsen, D. H. Baker, and J. C. McIver, “Low-Order Harmonic Interactions on AC/DC Systems,” *IEEE Trans. on Power Delivery*, Vol. 4, No. 1, January 1989, pp. 493-501.
14. S. G. Jalali and R. H. Lasseter, “A Study of Nonlinear Harmonic Interaction Between a Single Phase Line-Commutated Converter and a Power System,” *IEEE Trans. on Power Delivery*, Vol. 9, No. 3, July 1994, pp. 1616-1624.
15. D. Xia and G. T. Heydt, "Harmonic Power Flow Studies, Part I - Formulation and Solution, Part II - Implementation and Practical Application", *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-101, June 1982, pp. 1257-1270.
16. V. Sharma, R. J. Fleming, and L. Niekamp, “An Iterative Approach for Analysis of Harmonic Penetration in Power Transmission Networks,” *IEEE Trans. on Power Delivery*, Vol. 6, No. 4, October 1991, pp. 1698-1706.
17. M. Valcarcel and J. G. Mayordomo, “Harmonic Power Flow for Unbalanced Systems,” *IEEE Trans. on Power Delivery*, Vol. 8, No. 4, October 1993, pp. 2052-2059.
18. B. C. Smith, N. R. Watson, A. R. Wood, and J. Arrillaga, “A Newton Solution for the Harmonic Phasor Analysis of AC/DC Converters,” *IEEE Trans. on Power Delivery*, Vol. 11, No. 2, April 1996, pp. 965-971.
19. B. K. Perkins and M. R. Iravani, “Novel Calculation of HVDC Converter Harmonics by Linearization in the Time-Domain,” *IEEE Trans. on Power Delivery*, Vol. 12, No. 2, April 1997, pp. 867-873.
20. M. Grotzbach and Ch. Ried, “Investigation of AC/DC Converter Harmonics by an Analytical Based Time-Discrete Approach,” *IEEE Trans. on Power Delivery*, Vol. 12, No. 2, April 1997, pp. 874-880.
21. W. Xu, J. R. Marti, and H. W. Dommel, “Harmonic Analysis of Systems with Static Compensators,” *IEEE Trans. on Power Delivery*, Vol. 6, No. 1, February 1991, pp. 183-190.
22. L. J. Bohmann and R. H. Lasseter, “Harmonic Interactions in Thyristor Controlled Reactor Circuits,” *IEEE Trans. on Power Delivery*, Vol. 4, No. 3, July 1989, pp. 1919-1926.
23. J. Vittek and M. Y. Najjar, “Common Methodology for Steady State Harmonic Analysis of Inverters,” *IEEE Trans. on Power Delivery*, Vol. 10, No. 3, July 1995, pp. 1628-1634.
24. J. J. Rico, E. Acha, and T. J. E. Miller, “Harmonic Domain Modeling of Three Phase Thyristor-Controlled Reactors by Means of Switching Vectors and Discrete Convolutions,” *IEEE Trans. on Power Delivery*, Vol. 11, No. 3, July 1996, pp. 1678-1684.