Frequency-Domain Harmonic Analysis Methods

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Abstract: There are a large number of frequency-domain analysis methods that are in widespread use. The most popular of these are frequency scans, harmonic penetration, and harmonic power flow. Each of these techniques can be employed on a "per-phase" (positive or zero sequence) or "multi-phase" basis and each technique utilizes an admittance matrix system model developed from individual component-level models connected according to system topology. The development of this admittance matrix system model and the frequency-domain harmonic analysis algorithms that make use of it are the subjects of this chapter.

7.1. Introduction

The methodology employed in the development of admittance matrix models is based on multi-port network theory. Positive sequence admittance matrix models are developed from device-level two-port admittance (y) parameters. Multi-phase models are similarly developed from multi-port admittance parameters. Figure 7.1 shows a general multi-port model with appropriate terminal parameter definitions.

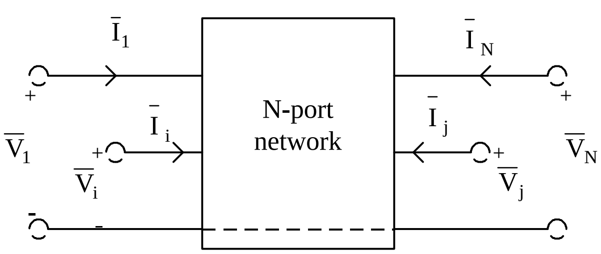

Figure 7.1. N-port Network

The arbitrary network of Figure 7.1 should, for simplicity, be limited to passive elements. It is possible, however, to include certain dependent sources provided that the dependence can be cast in an admittance relationship. Equation 7.1 describes the general network of Figure 7.1.

$$\begin{bmatrix} \bar{I}_1 \\ \\ \bar{I}_i \\ \bar{I}_j \\ \\ \bar{I}_N \end{bmatrix} = \begin{bmatrix} \bar{y}_{11} & \bar{y}_{1i} & \bar{y}_{1j} & \bar{y}_{1N} \\ \\ \bar{y}_{i1} & \bar{y}_{ii} & \bar{y}_{ij} & \bar{y}_{iN} \\ \bar{y}_{j1} & \bar{y}_{ji} & \bar{y}_{jj} & \bar{y}_{jN} \\ \\ \bar{y}_{N1} & \bar{y}_{Ni} & \bar{y}_{Nj} & \bar{y}_{NN} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \\ \bar{V}_i \\ \bar{V}_j \\ \\ \bar{V}_N \end{bmatrix} \quad (7.1)$$

The complex admittances (based on a known frequency) in (7.1) can be determined from the definitions of admittance parameters as shown in (7.2).

$$\bar{y}_{ij} = \left.\frac{\bar{I}_i}{\bar{V}_j}\right|_{\bar{I}_k=0;\ k=1..N,\ k\neq i} \quad (7.2)$$

For cases where a common node voltage reference point is available (as represented by the dashed line through the network in Figure 7.1), an advantageous simplification to (7.2) can be developed. The rules 1 and 2 as follows can be used to define any complex admittance y_{ij} in (7.1).

1: $\bar{y}_{ij} = -\sum(\text{all admittances connected between i and j})$
2: $\bar{y}_{ii} = \sum(\text{all admittances connected to i})$

These well-known procedures form the backbone of most admittance matrix formulation algorithms because they can be applied directly to a complete system. Formation of component-level admittance models is not required [1,2].

In some admittance matrix building algorithms, an incidence matrix [Q] is used to represent network connectivity. In this approach, a triple matrix product is used to form the system admittance matrix model as shown in (7.3) where the complex matrix $[Y_{prim}]$ consists of component-level admittance matrix models formed according to (7.2) or the simplifying rules 1 and 2.

$$\left[\bar{Y}_{sys}\right] = [Q]\left[\bar{Y}_{prim}\right][Q]^T \quad (7.3)$$

It should be noted that the procedure based on rules 1 and 2 is appropriate only when the common node voltage reference point is present. In general, this is true for either balanced or radial systems. Unbalanced nonradial systems should incorporate more generalized admittance matrix modeling techniques as described in [3].

As previously described, the modeling approach can be applied to either single-phase or multi-phase systems. For multi-phase systems, the rules 1 and 2 given previously must be considered to apply to 3x3 admittance matrices. The subscripts i and j, therefore, must be expanded to represent i+0, i+1, and i+2 and j+0, j+1, and j+2, respectively. The summations given in the rules apply so as to sum entries in the system admittance matrix with those in the 3x3 component-level models. If the approach of (7.3) is employed, the primitive admittance matrix will consist of 6x6 blocks provided that two or more components are not mutually coupled. The incidence matrix [Q] must be modified accordingly. References [4] and [5] provide implementation details for balanced or radial systems and more general unbalanced nonradial systems, respectively.

For harmonic analysis, the admittance matrix must be formulated at each frequency of interest. The matrix must be re-built from scratch; direct modifications to convert the system matrix from one frequency to another are not usually possible. In general, the matrix is re-built from the component level RLC parameters for circuit models for lines, transformers, and other power delivery equipment. The actual matrix construction procedures described in the previous paragraphs apply without modification.

7.2 Frequency Scan Analysis

Frequency scan analyses are used to characterize the response of a power delivery system as a function of frequency. The term "scan" arises from the systematic variation of frequency from some initial value f_0 to some final value f_F. The frequency scan analysis is conducted through repeated solutions of (7.1) with the admittance matrix formed for each frequency of interest. Equation (7.4) provides a clear "frequency-dependent" version of (7.1) where h is used to denote the harmonic frequency (in either Hz, rad/s, or pu).

$$\tilde{\bar{I}}(h) = \left[\bar{Y}(h)\right]\tilde{\bar{V}}(h) \qquad (7.4)$$

Two types of frequency scans are commonly performed. The first type is based on a single "current injection" into the power delivery system Y_{bus} model followed by a solution of (7.1). Assuming this injection takes place at node i in Figure 7.1, (7.1) can be solved to determine the voltages that are produced at each system node. If the current injection is assigned a value of $1/\underline{0}°$ (A or pu), the values of the determined voltages represent the driving point and transfer impedances as seen at node i. Because the Y_{bus} model contains only linear elements, linearity can be applied to scale the results obtained for the $1/\underline{0}°$ (A or pu) to any desired value. This scalability can be applied to estimate the harmonic voltages that will be produced at any network bus when a load that draws nonsinusoidal current is connected at the "injection" node. This technique is often used when assessing the potential impacts of new ASD or other harmonic-producing load and is very useful for identifying series and parallel resonances.

Varying the frequency used in the implementation and solution of 7.1 yields a series of impedance magnitudes and angles covering the range of frequency f_0-f_F. A plot of this series provides excellent visual indication of resonance conditions. Parallel resonances, which are associated with high impedances to current flows, appear as "peaks" in the plot. Series resonances, which are associated with low impedances to harmonic current flows, appear as "valleys" in the plot. A sample frequency scan is shown in Figure 7.2.

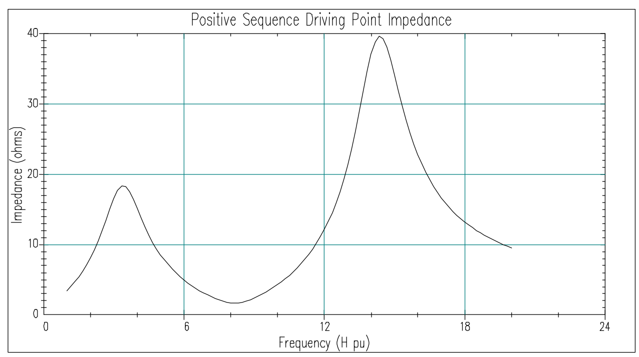

Figure 7.2. Typical Frequency (Impedance) Scan

The second type of scan is conducted in the same manner as described previously, except that a $1/\underline{0}°$ voltage (V or pu) is connected to one node in the network. Equation (7.1) is then solved for all other voltages in the network. The resulting voltages represent the voltage transfer functions to all other nodes in the system model. While similar in technique to the previously described frequency scan, the procedure of applying a $1/\underline{0}°$ voltage source is more typically called a "voltage transfer function" analysis to allow the term "frequency scan" to be associated with driving point and transfer impedances determined by injecting a known current into a node.

The voltage transfer function analysis is useful for investigating the effects of background harmonics. The terminology "background harmonics" refers to the harmonic voltage distortion that may be present at the terminals of any network equivalent. As with the impedance (frequency) scan, a plot of the voltage transfer function as a function of frequency can be used to reveal potential problems. Peaks in the plot indicate frequencies at which voltages will be amplified and valleys indicate frequencies at which voltages will be attenuated. An example plot of a voltage transfer function is shown in Figure 7.3.

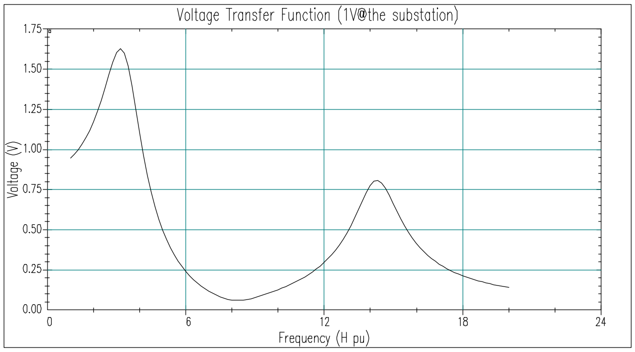

Figure 7.3. Voltage Transfer Function Plot

As previously mentioned, the admittance matrix system model can be formed based on sequence networks or phase-variable networks. Multiphase systems can be handled without modification. Because the frequency scans described previously (both types) are only modifications of this admittance model, the procedures are equally applicable to sequence- and phase-variable models. It should be noted, however, that the phase angles of the voltage or current injections are important. In phase variables, a three-phase positive sequence scan would be conducted by injecting a current vector of $[1/\underline{0}^\circ \; 1/\underline{-120}^\circ \; 1/\underline{120}^\circ]^T$ (A or pu) into a three-phase bus. A zero sequence scan would be conducted in the same manner except all phase angles would be equal (typically 0). Of course, a single-phase scan would involve only an injection into one node of a three-phase bus. The appropriate injections into sequence variable-based admittance models can be determined by applying the symmetrical component transformation to the values given for the phase-variable injections.

7.3 Current Source Methods

Current source (or current injection) methods are the most popular forms of harmonic analyses. The methods all make use of the admittance model as described in section 7.1. The analysis procedure is similar to the frequency scan analysis for current injections in that nonlinear loads are represented using harmonic current spectra of known magnitude and phase. More specifically, nonlinear loads are represented using a summation of currents where each entry in the sum corresponds to a term of known frequency in the Fourier series representation of the load current. Taken collectively, this sum is often referred to as a "vector." With these spectral vectors (magnitude and angle at each harmonic of interest) known for each load, the analysis approach proceeds along the following series of steps:

Step 1. Formulate the system admittance matrix model of the power delivery system including contributions for all sources and linear loads. The frequency should be consistent with one of those in the Fourier series current vectors for the nonlinear loads.

Step 2. Construct the current injection vector in (7.1) by extracting the term of the appropriate frequency (which must match the frequency used in the admittance matrix model construction) from each of the nonlinear load harmonic current vectors.

Step 3. Solve (7.1) to determine the voltages at all network buses. The frequency associated with these phasor voltages is the same as that used in the construction of the admittance model.

The steps 1-3 begin at the lowest frequency represented in any of the load current harmonic vectors and repeat for each frequency in all of the nonlinear load models. It should be clear that not all nodes will posses harmonic load current injections at all frequencies; some loads inject 5th, 7th, 11th, 13th, etc., while others inject 3rd, 5th, 7th, 9th, etc. For the case where a nonlinear load does not inject a current at a particular frequency (but another nonlinear load does), it is a simple matter to force the injection to a zero value at that load bus and continue the solution of (7.1) at the frequency of interest.

The results of an analysis conducted using the current injection method is a collection (again, often referred to as a vector) of harmonic voltages for each bus in the system. Due to the linear nature of the problem (all nonlinearities have been represented as current injections), superposition can be applied. Therefore, the terms in each voltage vector correspond to the Fourier coefficients of the time-domain voltage. These spectra (magnitude only; phase is usually not shown) are often shown graphically as in Figure 7.4. If desired, the time-domain waveform can be easily constructed from the voltage spectrum at each network bus.

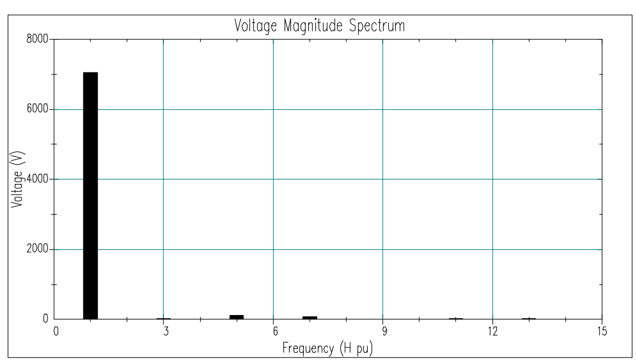

Figure 7.4. Example Voltage Magnitude Spectrum

The preceding paragraphs have described the procedure in general terms. In practice, there are a number of modifications that are used and that can, in some circumstance, produce markedly improved results. The most pronounced modification to the general procedure is the use of phase information in each nonlinear load harmonic current vector.

In studies where only a single nonlinear load is present (or a single nonlinear load dominants all others), the phase angles for each harmonic current phasor are not important. In systems with multiple nonlinear loads, however, neglecting harmonic current phase angles in load models can lead to inaccurate results. For systems with multiple nonlinear loads, harmonic currents at each frequency may be additive or subtractive, so voltage harmonics determined using steps 1-3 could be either over- or under-conservative for any particular harmonic.

Another enhancement that is often made is to include the effects of the fundamental frequency terminal voltage on the harmonic currents generated by a nonlinear load [6]. As an example, consider the voltage and current waveforms shown in Figure 7.5. For the voltage given in (7.5), the load current is as given in (7.6).

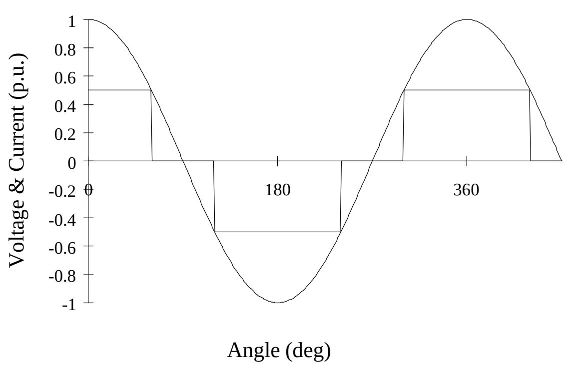

Figure 7.5. Example Voltage and Nonlinear Load Current Waveforms

$$v_{an}(t) = \sqrt{2}V_{rms}\cos(\omega t) \qquad (7.5)$$

$$\begin{aligned} i_a(t) = &\frac{2\sqrt{3}}{\pi}\cos(\omega t) \\ &-\frac{2\sqrt{3}}{5\pi}\cos(5\omega t) \\ &+\frac{2\sqrt{3}}{7\pi}\cos(7\omega t) \\ &\ldots \end{aligned} \qquad (7.6)$$

However, there can be significant differences in fundamental frequency bus voltage phase angles in actual systems. Modifying (7.5) to include an arbitrary phase angle δ as shown in (7.7) leads to the modification of (7.6) as shown in (7.8). Notice that the fundamental voltage angle is multiplied by "n" in the Fourier series of the current waveform, where n is the harmonic order of each term.

$$v'_{an}(t) = \sqrt{2}V_{rms}\cos(\omega t + \delta) \qquad (7.7)$$

$$\begin{aligned} i_a(t) = &\frac{2\sqrt{3}}{\pi}\cos(\omega t + \delta) \\ &-\frac{2\sqrt{3}}{5\pi}\cos(5\omega t + 5\delta) \\ &+\frac{2\sqrt{3}}{7\pi}\cos(7\omega t + 7\delta) \\ &\ldots \end{aligned} \qquad (7.8)$$

Note that this same corrective action of nδ is the basis for the use of 12 pulse (and higher) drives. In these higher-order drive systems, significant harmonic cancellation can be obtained for certain harmonics due only to the nδ correction.

One limitation of the current source method is the validity of the harmonic current vector representation of the nonlinear loads. Past experience has shown that this representation is valid for most nonlinear loads up to the point where the load terminal voltage distortion exceeds 10%. However, ongoing research in this field indicates a clear impact of terminal voltage distortion on certain nonlinear load harmonics, especially those produced by popular ASDs [7].

In addition, the current source method is limited to "snapshot" scenarios where the harmonic current source vectors represent very specific load patterns. It is well known that many nonlinear loads, including ASDs, produce markedly different harmonic currents depending on load level. Figures 7.6 (a) and (b) show the line current waveforms (and the associated harmonic magnitude spectra) drawn by a 250hp dc drive for (a) light load (spinning) and (b) fully loaded. It is difficult to capture the range of harmonics associated with these two line current conditions (and all loading points in between) without a large number of simulations using the current source methods. Further complicating the problem is the variation in fundamental frequency terminal voltage phase angle that accompanies changing load conditions.

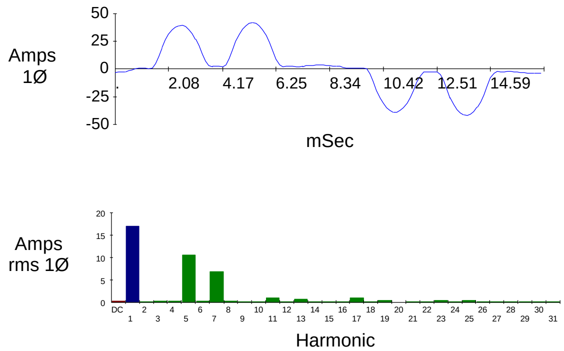

Figure 7.6(a). Lightly-Loaded DC Drive Line Current

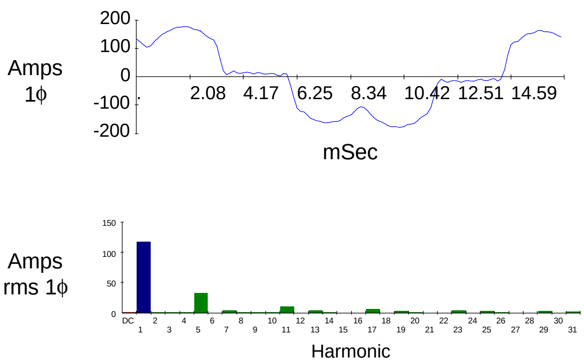

Figure 7.6(b). Fully-Loaded DC Drive Line Current

7.4 Harmonic Power Flow

The deficiencies in the current source method can be partially overcome using a technique that has come to be known as "harmonic power flow" or HPF. HPF algorithms combine the current source methodology with a conventional power flow algorithm. There are two basic variations of HPFs that find widespread use, and they are described as follows:

1. A fundamental frequency power flow solution is executed using a linear model for all power delivery equipment and loads, and the resultant fundamental frequency load terminal voltages are used to "adjust" nonlinear load harmonic current vectors (as shown in (7.8)) automatically without additional user action. The harmonic current vector is still required to be known for each load (as was the case for the current source methods).

2. All (or some) nonlinear load harmonic current spectra are represented in the form of (7.9) where $C_1,\ldots,C_M$ represent M control variables that are used to control various load parameters (such as shaft speed in a motor drive) and the phasor voltages $V_1,\ldots,V_N$ represent the harmonic voltage phasors at the load terminals. Nonlinear load representations of this form are used in conjunction with (7.1), where specific instances of (7.1) are required for each frequency included in (7.8), to form a complete mathematical model of the system. The entire set of equations is then solved iteratively using either Newton or Gaussian methods. Linear loads may be represented with a combination of impedances or with a constant power (P+jQ) model.

$$\begin{bmatrix} \bar{I}_1 \\ \vdots \\ \bar{I}_N \end{bmatrix} = \begin{bmatrix} f_1\left(\bar{V}_1,\ldots,\bar{V}_N,C_1,\ldots,C_M\right) \\ \vdots \\ f_N\left(\bar{V}_1,\ldots,\bar{V}_N,C_1,\ldots,C_M\right) \end{bmatrix} \tag{7.9}$$

The first HPF version is a relatively simple extension of the current source method. The same limitations apply, and the only advantage is the automatic "correction" for fundamental frequency terminal voltage. Due to this relatively minor improvement, this first improvement is considered by many not to be a true HPF.

The second HPF version is an extremely complex and powerful technique. The system model is formed as described for the current source method, but the loads can be modeled in an almost arbitrarily complex manner depending on the amount of detail required to obtain the level of accuracy desired. When a closed-form solution for the nonlinear load current harmonics can be obtained as a function of voltage (including voltage harmonics) and control parameters, it is possible to represent the load harmonics directly in the frequency domain as shown in (7.9). In many cases, this closed-form solution can not be obtained and a combination time- and frequency-domain technique is employed.

The so-called hybrid HPFs utilize a power delivery system model in the form of (7.1) for each harmonic, but represent the nonlinear loads with time-domain differential equations [8]. Given an initial estimate of the network voltages, the load models are simulated (they can be decoupled if necessary) until steady-state is reached. A new harmonic current vector is then created from the steady-state current for each nonlinear load. These current injections are then used as described for the current source method to obtain an updated terminal voltage (including harmonics). The procedure continues until the frequency domain network model converges with all nonlinear load models in steady-state.

The hybrid methods are by far the most powerful, but they are also the most complex. It is possible to represent converter controls, for example, in great detail so as to account for virtually every possible harmonic scenario. With this capability, however, comes the requirement that the user must have the data and expertise required. More often than not, this is not the case. Detailed harmonic studies involving complex converter controls or widely-varying load patterns, therefore, are often best analyzed using complete time-domain models that are simulated using a transient analysis program such as the EMTP.

7.5 Conclusions

In this chapter, the most popular frequency-domain harmonic analysis methods have been presented. While the approach presented here concentrates on admittance matrix modeling, it is equally possible to utilize advance impedance matrix models in conjunction with each analysis type presented. Each of the methods has found significant practical application as illustrated in the case studies found throughout this tutorial. It is, however, always up to the analyst to weigh the costs (improved complexity) and benefits (increased accuracy) of any given method for the problem at hand.

7.6 References

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